

***GRE*[®] Mathematics Test** Practice Book

This practice book contains

- one actual, full-length *GRE*[®] Mathematics Test
- test-taking strategies

Become familiar with

- test structure and content
- test instructions and answering procedures

Compare your practice test results with the performance of those who took the test at a GRE administration.



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Test takers with disabilities or health-related needs who need test preparation materials in an alternate format should contact the ETS Office of Disability Services at **stassd@ets.org**. For additional information, visit **www.ets.org/gre/disabilities**.

Overview

The GRE® Mathematics Test consists of approximately 66 multiple-choice questions drawn from courses commonly offered at the undergraduate level. Testing time is 2 hours and 50 minutes; there are no separately-timed sections.

This publication provides a comprehensive overview of the GRE Mathematics Test to help you get ready for test day. It is designed to help you:

- Understand what is being tested
- Gain familiarity with the question types
- Review test-taking strategies
- Understand scoring
- Practice taking the test

To learn more about the GRE Subject Tests, visit www.ets.org/gre.

Test Content

Approximately 50 percent of the questions involve calculus and its applications — subject matter that is assumed to be common to the backgrounds of almost all mathematics majors. About 25 percent of the questions in the test are in elementary algebra, linear algebra, abstract algebra, and number theory. The remaining questions deal with other areas of mathematics currently studied by undergraduates in many institutions.

The following content descriptions may assist students in preparing for the test. The percents given are estimates; actual percents will vary somewhat from one edition of the test to another.

Calculus (50%)

- Material learned in the usual sequence of elementary calculus courses — differential and integral calculus of one and of several variables — including calculus-based applications and connections with coordinate geometry, trigonometry, differential equations, and other branches of mathematics

Algebra (25%)

- Elementary algebra: basic algebraic techniques and manipulations acquired in high school and used throughout mathematics
- Linear algebra: matrix algebra, systems of linear equations, vector spaces, linear transformations, characteristic polynomials, and eigenvalues and eigenvectors
- Abstract algebra and number theory: elementary topics from group theory, theory of rings and modules, field theory, and number theory

Additional Topics (25%)

- Introductory real analysis: sequences and series of numbers and functions, continuity, differentiability and integrability, and elementary topology of \mathbb{R} and \mathbb{R}^n
- Discrete mathematics: logic, set theory, combinatorics, graph theory, and algorithms
- Other topics: general topology, geometry, complex variables, probability and statistics, and numerical analysis

The above descriptions of topics covered in the test **should not be considered exhaustive**; it is necessary to understand many other related concepts. Prospective test takers should be aware that questions requiring no more than a good precalculus background may be quite challenging; such questions can be among the most difficult questions on the test. In general, the questions are intended not only to test recall of information, but also to assess the understanding of fundamental concepts and the ability to apply those concepts in various situations.

Preparing for the Test

GRE Subject Test questions are designed to measure skills and knowledge gained over a long period of time. Although you might increase your scores to some extent through preparation a few weeks or months before you take the test, last minute cramming is unlikely to be of further help.

The following information may be helpful.

- A general review of your college courses is probably the best preparation for the test. However, the test covers a broad range of subject matter, and no one is expected to be familiar with the content of every question.
- Use the practice test to become familiar with the types of questions in the GRE Mathematics Test, taking note of the directions. If you understand the directions before you take the test, you will have more time during the test to focus on the questions themselves.

Test-Taking Strategies

The questions in the practice test in this book illustrate the types of multiple-choice questions in the test. When you take the actual test, you will mark your answers on a separate machine-scorable answer sheet.

Following are some general test-taking strategies you may want to consider.

- Read the test directions carefully, and work as rapidly as you can without being careless. For each question, choose the best answer from the available options.
- All questions are of equal value; do not spend time pondering individual questions you find extremely difficult or unfamiliar.
- You may want to work through the test quite rapidly, first answering only the questions about which you feel confident, then going back and answering questions that require more thought, and concluding with the most difficult questions if there is time.
- If you decide to change an answer, make sure you completely erase it and fill in the oval corresponding to your desired answer.
- Questions for which you mark no answer or more than one answer are not counted in scoring.
- Your score will be determined by subtracting one-fourth the number of incorrect answers

from the number of correct answers. It is unlikely that pure guessing will raise your score; it may lower your score. However, if you have some knowledge of a question and are able to rule out one or more of the answer choices as incorrect, your chances of selecting the correct answer are improved, and answering such questions will likely improve your score.

- Record all answers on your answer sheet. Answers recorded in your test book will not be counted.
- Do not wait until the last five minutes of a testing session to record answers on your answer sheet.

What Your Scores Mean

Your raw score — that is, the number of questions you answered correctly minus one-fourth of the number you answered incorrectly — is converted to the scaled score that is reported. This conversion ensures that a scaled score reported for any edition of a GRE Mathematics Test is comparable to the same scaled score earned on any other edition of the test. Thus, equal scaled scores indicate essentially equal levels of performance regardless of the test edition taken.

GRE Mathematics Test scores are reported on a 200 to 990 score scale in ten-point increments.

Test scores should be compared only with other scores on the GRE Mathematics Test. For example, a 680 on the GRE Mathematics Test is not equivalent to a 680 on the GRE Physics Test.

Taking the Practice Test

The practice test begins on page 7. The total time that you should allow for the practice test is 2 hours and 50 minutes. An answer sheet is provided for you to mark your answers to the test questions.

It is best to take the practice test under timed conditions. Find a quiet place to take the test and make sure you have a minimum of 2 hours and 50 minutes available.

To simulate how the administration will be conducted at the test center, print the answer sheet (pages 65 and 66). Then go to page 62 and follow the instructions for completing the identification areas of the answer sheet. When you are ready to begin the test, note the time and begin marking your answers on the answer sheet. Stop working on the test when 2 hours and 50 minutes have elapsed.

Scoring the Practice Test

The worksheet on page 63 lists the correct answers to the questions. Columns are provided for you to mark whether you chose the correct (C) answer or an incorrect (I) answer to each question. Draw a line across any question you omitted, because it is not counted in the scoring.

At the bottom of the page, enter the total number correct and the total number incorrect. Divide the total incorrect by 4 and subtract the resulting number from the total correct. Then round the result to the nearest whole number. This will give you your raw score. Use the score conversion table on page 64 to find the scaled score that corresponds to your raw score.

Example: Suppose you chose the correct answers to 46 questions and incorrect answers to 9. Dividing 9 by 4 yields 2.25. Subtracting 2.25 from 46 equals 43.75, which is rounded to 44. The raw score of 44 corresponds to a scaled score of 770.

Evaluating Your Performance

Now that you have scored your test, you may wish to compare your performance with the performance of others who took this test.

The data in the worksheet on page 63 are based on the performance of a sample of the test takers who took the GRE Mathematics Test in October 2012. This sample was selected to represent the total population of GRE Mathematics Test examinees tested between July 1, 2011, and June 30, 2014.

The numbers in the column labeled “P+” on the worksheet indicate the percentages of examinees in this sample who answered each question correctly.

You may use these numbers as a guide for evaluating your performance on each test question.

Interpretive data based on the scores earned by test takers in a recent three-year period are available in Table 2, *Subject Tests Total Score Interpretive Data Used on Score Reports*, at www.ets.org/gre/subject/scores/understand. The table shows selected scaled scores and the corresponding percentage of test takers who received lower scores. Note that these interpretive data are updated annually and appear on GRE score reports. To compare yourself with this population, look at the percentage next to the scaled score you earned on the practice test. If the scaled score you earned does not appear in the table, look at the scaled scores (and corresponding percentages) above and below the score you earned to assess your performance.

It is important to realize that the conditions under which you tested yourself were not exactly the same as those you will encounter at a test center. It is impossible to predict how different test-taking conditions will affect test performance, and this is only one factor that may account for differences between your practice test scores and your actual test scores. By comparing your performance on the practice test with the performance of other individuals who took GRE Mathematics Test, however, you will be able to determine your strengths and weaknesses and can then plan a program of study to prepare yourself for taking the GRE Mathematics Test under standard conditions.





FORM GR1268

68

GRADUATE RECORD EXAMINATIONS®

MATHEMATICS TEST

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MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is best and then completely fill in the corresponding space on the answer sheet.

Computation and scratch work may be done in this test book.

In this test:

- (1) All logarithms with an unspecified base are natural logarithms, that is, with base e .
- (2) The symbols \mathbb{Z} , \mathbb{Q} , \mathbb{R} , and \mathbb{C} denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.

1. $\lim_{x \rightarrow 0} \frac{\cos(3x) - 1}{x^2} =$

- (A) $\frac{9}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{3}$ (D) $-\frac{3}{2}$ (E) $-\frac{9}{2}$

2. What is the area of an equilateral triangle whose inscribed circle has radius 2 ?

- (A) 12 (B) 16 (C) $12\sqrt{3}$ (D) $16\sqrt{3}$ (E) $4(3 + 2\sqrt{2})$

3. $\int_{e^{-3}}^{e^{-2}} \frac{1}{x \log x} dx =$

- (A) 1 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\log\left(\frac{2}{3}\right)$ (E) $\log\left(\frac{3}{2}\right)$
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

4. Let V and W be 4-dimensional subspaces of a 7-dimensional vector space X . Which of the following CANNOT be the dimension of the subspace $V \cap W$?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
-

5. Sofia and Tess will each randomly choose one of the 10 integers from 1 to 10. What is the probability that neither integer chosen will be the square of the other?

- (A) 0.64 (B) 0.72 (C) 0.81 (D) 0.90 (E) 0.95
-

6. Which of the following shows the numbers $2^{1/2}$, $3^{1/3}$, and $6^{1/6}$ in increasing order?

(A) $2^{1/2} < 3^{1/3} < 6^{1/6}$

(B) $6^{1/6} < 3^{1/3} < 2^{1/2}$

(C) $6^{1/6} < 2^{1/2} < 3^{1/3}$

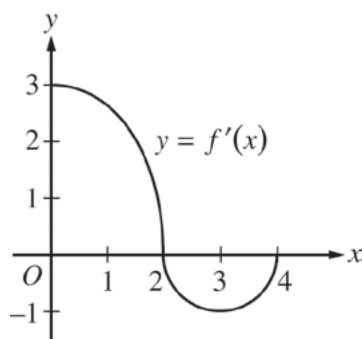
(D) $3^{1/3} < 2^{1/2} < 6^{1/6}$

(E) $3^{1/3} < 6^{1/6} < 2^{1/2}$

GO ON TO THE NEXT PAGE.



SCRATCH WORK



7. The figure above shows the graph of the derivative f' of a function f , where f is continuous on the interval $[0, 4]$ and differentiable on the interval $(0, 4)$. Which of the following gives the correct ordering of the values $f(0)$, $f(2)$, and $f(4)$?
- (A) $f(0) < f(2) < f(4)$
- (B) $f(0) < f(4) = f(2)$
- (C) $f(0) < f(4) < f(2)$
- (D) $f(4) = f(2) < f(0)$
- (E) $f(4) < f(0) < f(2)$
-
8. Which of the following is NOT a group?
- (A) The integers under addition
- (B) The nonzero integers under multiplication
- (C) The nonzero real numbers under multiplication
- (D) The complex numbers under addition
- (E) The nonzero complex numbers under multiplication
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

9. Let g be a continuous real-valued function defined on \mathbb{R} with the following properties.

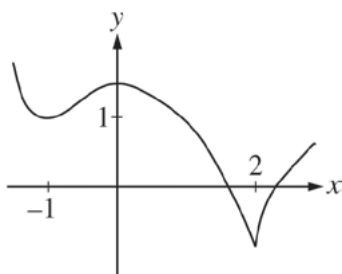
$$g'(0) = 0$$

$$g''(-1) > 0$$

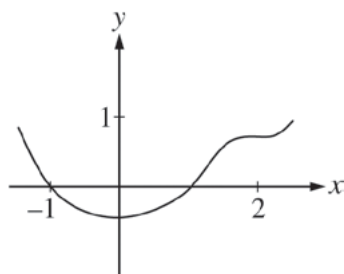
$$g''(x) < 0 \text{ if } 0 < x < 2.$$

Which of the following could be part of the graph of g ?

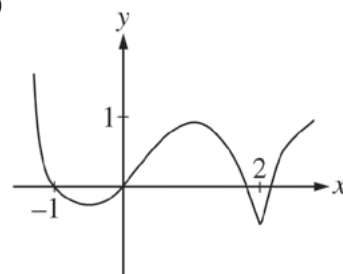
(A)



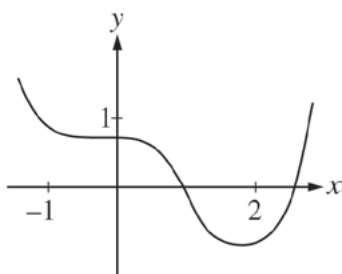
(B)



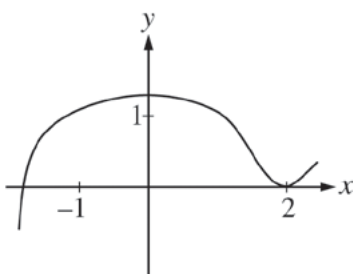
(C)



(D)



(E)



$$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-3)^2 + y^2}$$

10. In the xy -plane, the set of points whose coordinates satisfy the equation above is

- (A) a line (B) a circle (C) an ellipse (D) a parabola (E) one branch of a hyperbola

GO ON TO THE NEXT PAGE.



SCRATCH WORK



11. The region bounded by the curves $y = x$ and $y = x^2$ in the first quadrant of the xy -plane is rotated about the y -axis. The volume of the resulting solid of revolution is

- (A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) $\frac{3\pi}{2}$
-

12. For which integers n such that $3 \leq n \leq 11$ is there only one group of order n (up to isomorphism) ?

- (A) For no such integer n
(B) For 3, 5, 7, and 11 only
(C) For 3, 5, 7, 9, and 11 only
(D) For 4, 6, 8, and 10 only
(E) For all such integers n
-

13. If f is a continuously differentiable real-valued function defined on the open interval $(-1, 4)$ such that $f(3) = 5$ and $f'(x) \geq -1$ for all x , what is the greatest possible value of $f(0)$?

- (A) 3 (B) 4 (C) 5 (D) 8 (E) 11
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

14. Suppose g is a continuous real-valued function such that $3x^5 + 96 = \int_c^x g(t) dt$ for each $x \in \mathbb{R}$, where c is a constant. What is the value of c ?
- (A) -96 (B) -2 (C) 4 (D) 15 (E) 32
-

15. Let S , T , and U be nonempty sets, and let $f: S \rightarrow T$ and $g: T \rightarrow U$ be functions such that the function $g \circ f: S \rightarrow U$ is one-to-one (injective). Which of the following must be true?
- (A) f is one-to-one.
(B) f is onto.
(C) g is one-to-one.
(D) g is onto.
(E) $g \circ f$ is onto.
-

16. Suppose A , B , and C are statements such that C is true if exactly one of A and B is true. If C is false, which of the following statements must be true?
- (A) If A is true, then B is false.
(B) If A is false, then B is false.
(C) If A is false, then B is true.
(D) Both A and B are true.
(E) Both A and B are false.
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

17. Which of the following equations has the greatest number of real solutions?

(A) $x^3 = 10 - x$

(B) $x^2 + 5x - 7 = x + 8$

(C) $7x + 5 = 1 - 3x$

(D) $e^x = x$

(E) $\sec x = e^{-x^2}$

18. Let f be the function defined by $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ for all x such that $-1 < x < 1$. Then $f'(x) =$

(A) $\frac{1}{1-x}$

(B) $\frac{x}{1-x}$

(C) $\frac{1}{1+x}$

(D) $\frac{x}{1+x}$

(E) 0

19. If z is a complex variable and \bar{z} denotes the complex conjugate of z , what is $\lim_{z \rightarrow 0} \frac{(\bar{z})^2}{z^2}$?

(A) 0

(B) 1

(C) i

(D) ∞

(E) The limit does not exist.

GO ON TO THE NEXT PAGE.



SCRATCH WORK

20. Let g be the function defined by $g(x) = e^{2x+1}$ for all real x . Then $\lim_{x \rightarrow 0} \frac{g(g(x)) - g(e)}{x} =$
- (A) $2e$ (B) $4e^2$ (C) e^{2e+1} (D) $2e^{2e+1}$ (E) $4e^{2e+2}$
-

21. What is the value of $\int_{-\pi/4}^{\pi/4} (\cos t + \sqrt{1+t^2} \sin^3 t \cos^3 t) dt$?
- (A) 0 (B) $\sqrt{2}$ (C) $\sqrt{2} - 1$ (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{\sqrt{2}-1}{2}$
-

22. What is the volume of the solid in xyz -space bounded by the surfaces $y = x^2$, $y = 2 - x^2$, $z = 0$, and $z = y + 3$?
- (A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) $\frac{32}{3}$ (D) $\frac{104}{105}$ (E) $\frac{208}{105}$
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

23. Let $(\mathbb{Z}_{10}, +, \cdot)$ be the ring of integers modulo 10, and let S be the subset of \mathbb{Z}_{10} represented by $\{0, 2, 4, 6, 8\}$. Which of the following statements is FALSE?
- (A) $(S, +, \cdot)$ is closed under addition modulo 10.
 - (B) $(S, +, \cdot)$ is closed under multiplication modulo 10.
 - (C) $(S, +, \cdot)$ has an identity under addition modulo 10.
 - (D) $(S, +, \cdot)$ has no identity under multiplication modulo 10.
 - (E) $(S, +, \cdot)$ is commutative under addition modulo 10.
-

24. Consider the system of linear equations

$$\begin{aligned}w + 3x + 2y + 2z &= 0 \\w + 4x + y &= 0 \\3w + 5x + 10y + 14z &= 0 \\2w + 5x + 5y + 6z &= 0\end{aligned}$$

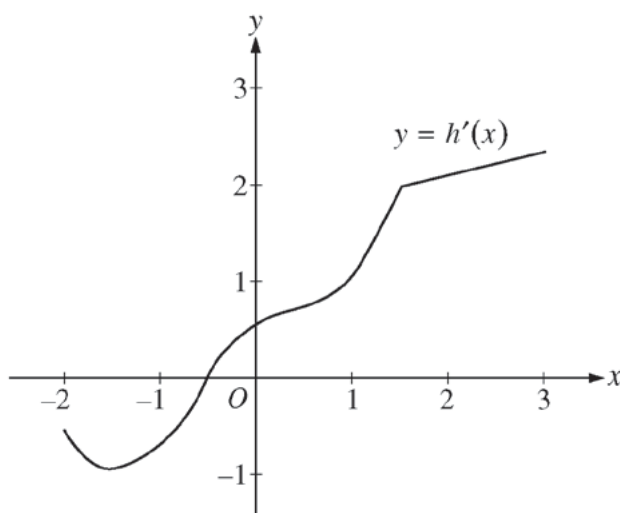
with solutions of the form (w, x, y, z) , where w, x, y , and z are real. Which of the following statements is FALSE?

- (A) The system is consistent.
 - (B) The system has infinitely many solutions.
 - (C) The sum of any two solutions is a solution.
 - (D) $(-5, 1, 1, 0)$ is a solution.
 - (E) Every solution is a scalar multiple of $(-5, 1, 1, 0)$.
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK



25. The graph of the derivative h' is shown above, where h is a real-valued function. Which of the following open intervals contains a value c for which the point $(c, h(c))$ is an inflection point of h ?

- (A) $(-2, -1)$ (B) $(-1, 0)$ (C) $(0, 1)$ (D) $(1, 2)$ (E) $(2, 3)$

$$3x \equiv 5 \pmod{11}$$

$$2y \equiv 7 \pmod{11}$$

26. If x and y are integers that satisfy the congruences above, then $x + y$ is congruent modulo 11 to which of the following?

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

27. $(1 + i)^{10} =$

- (A) 1 (B) i (C) 32 (D) $32i$ (E) $32(i + 1)$

GO ON TO THE NEXT PAGE.



SCRATCH WORK

28. Let f be a one-to-one (injective), positive-valued function defined on \mathbb{R} . Assume that f is differentiable at $x = 1$ and that in the xy -plane the line $y - 4 = 3(x - 1)$ is tangent to the graph of f at $x = 1$. Let g be the function defined by $g(x) = \sqrt{x}$ for $x \geq 0$. Which of the following is FALSE?

(A) $f'(1) = 3$

(B) $(f^{-1})'(4) = \frac{1}{3}$

(C) $(fg)'(1) = 5$

(D) $(g \circ f)'(1) = \frac{1}{2}$

(E) $(g \circ f)(1) = 2$

29. A tree is a connected graph with no cycles. How many nonisomorphic trees with 5 vertices exist?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

30. For what positive value of c does the equation $\log x = cx^4$ have exactly one real solution for x ?

(A) $\frac{1}{4e}$ (B) $\frac{1}{4e^4}$ (C) $\frac{e^4}{4}$ (D) $\frac{4}{e^{1/4}}$ (E) $4e^{1/4}$

GO ON TO THE NEXT PAGE.



SCRATCH WORK

31. Of the numbers 2, 3, and 5, which are eigenvalues of the matrix $\begin{pmatrix} 3 & 5 & 3 \\ 1 & 7 & 3 \\ 1 & 2 & 8 \end{pmatrix}$?

- (A) None (B) 2 and 3 only (C) 2 and 5 only (D) 3 and 5 only (E) 2, 3, and 5
-

32. $\frac{d}{dx} \int_{x^3}^{x^4} e^{t^2} dt =$

- (A) $e^{x^6} (e^{x^8 - x^6} - 1)$ (B) $4x^3 e^{x^8}$ (C) $\frac{1}{\sqrt{1 - e^{x^2}}}$ (D) $\frac{e^{x^2}}{x^2} - 1$ (E) $x^2 e^{x^6} (4x e^{x^8 - x^6} - 3)$
-

33. What is the 19th derivative of $\frac{x-1}{e^x}$?

- (A) $(18 - x)e^{-x}$
 (B) $(19 - x)e^{-x}$
 (C) $(20 - x)e^{-x}$
 (D) $(x - 19)e^{-x}$
 (E) $(x - 20)e^{-x}$
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

34. Which of the following statements about the real matrix shown above is FALSE?

- (A) A is invertible.
- (B) If $\mathbf{x} \in \mathbb{R}^5$ and $A\mathbf{x} = \mathbf{x}$, then $\mathbf{x} = \mathbf{0}$.
- (C) The last row of A^2 is $(0 \ 0 \ 0 \ 0 \ 25)$.
- (D) A can be transformed into the 5×5 identity matrix by a sequence of elementary row operations.
- (E) $\det(A) = 120$

35. In xyz -space, what are the coordinates of the point on the plane $2x + y + 3z = 3$ that is closest to the origin?

- (A) $(0, 0, 1)$
- (B) $\left(\frac{3}{7}, \frac{3}{14}, \frac{9}{14}\right)$
- (C) $\left(\frac{7}{15}, \frac{8}{15}, \frac{1}{15}\right)$
- (D) $\left(\frac{5}{6}, \frac{1}{3}, \frac{1}{3}\right)$
- (E) $\left(1, 1, \frac{1}{3}\right)$

GO ON TO THE NEXT PAGE.



SCRATCH WORK

36. Suppose S is a nonempty subset of \mathbb{R} . Which of the following is necessarily true?

- (A) For each $s, t \in S$, there exists a continuous function f mapping $[0, 1]$ into S with $f(0) = s$ and $f(1) = t$.
 - (B) For each $u \notin S$, there exists an open subset U of \mathbb{R} such that $u \in U$ and $U \cap S = \emptyset$.
 - (C) $\{v \in S : \text{there exists an open subset } V \text{ of } \mathbb{R} \text{ with } v \in V \subseteq S\}$ is an open subset of \mathbb{R} .
 - (D) $\{w \notin S : \text{there exists an open subset } W \text{ of } \mathbb{R} \text{ with } w \in W \text{ and } W \cap S = \emptyset\}$ is a closed subset of \mathbb{R} .
 - (E) S is the intersection of all closed subsets of \mathbb{R} that contain S .
-

37. Let V be a finite-dimensional real vector space and let P be a linear transformation of V such that $P^2 = P$. Which of the following must be true?

- I. P is invertible.
 - II. P is diagonalizable.
 - III. P is either the identity transformation or the zero transformation.
- (A) None (B) I only (C) II only (D) III only (E) II and III
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

38. The maximum number of acute angles in a convex 10-gon in the Euclidean plane is

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
-

39. Consider the following algorithm, which takes an input integer $n > 2$ and prints one or more integers.

```
input (n)
set i = 1
while i < n
  begin
    replace i by i + 1
    set k = n
    while k ≥ i
      begin
        if i = k then print(i)
        replace k by k - 1
      end
    end
  end
end
```

If the input integer is 88, what integers will be printed?

- (A) Only the integer 2
(B) Only the integer 88
(C) Only the divisors of 88 that are greater than 1
(D) The integers from 2 to 88 in increasing order
(E) The integers from 88 to 2 in decreasing order
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

40. Let S be the set of all functions $f: \mathbb{R} \rightarrow \mathbb{R}$. Consider the two binary operations $+$ and \circ on S defined as pointwise addition and composition of functions, as follows.

$$(f + g)(x) = f(x) + g(x)$$

$$(f \circ g)(x) = f(g(x))$$

Which of the following statements are true?

- I. \circ is commutative.
 - II. $+$ and \circ satisfy the left distributive law $f \circ (g + h) = (f \circ g) + (f \circ h)$.
 - III. $+$ and \circ satisfy the right distributive law $(g + h) \circ f = (g \circ f) + (h \circ f)$.
- (A) None (B) II only (C) III only (D) II and III only (E) I, II, and III

41. Let ℓ be the line that is the intersection of the planes $x + y + z = 3$ and $x - y + z = 5$ in \mathbb{R}^3 . An equation of the plane that contains $(0, 0, 0)$ and is perpendicular to ℓ is

- (A) $x - z = 0$
- (B) $x + y + z = 0$
- (C) $x - y - z = 0$
- (D) $x + z = 0$
- (E) $x + y - z = 0$

GO ON TO THE NEXT PAGE.



SCRATCH WORK

42. Let \mathbb{Z}^+ be the set of positive integers and let d be the metric on \mathbb{Z}^+ defined by

$$d(m, n) = \begin{cases} 0 & \text{if } m = n \\ 1 & \text{if } m \neq n \end{cases}$$

for all $m, n \in \mathbb{Z}^+$. Which of the following statements are true about the metric space (\mathbb{Z}^+, d) ?

- I. If $n \in \mathbb{Z}^+$, then $\{n\}$ is an open subset of \mathbb{Z}^+ .
 - II. Every subset of \mathbb{Z}^+ is closed.
 - III. Every real-valued function defined on \mathbb{Z}^+ is continuous.
- (A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

43. A curve in the xy -plane is given parametrically by

$$\begin{aligned} x &= t^2 + 2t \\ y &= 3t^4 + 4t^3 \end{aligned}$$

for all $t > 0$. The value of $\frac{d^2y}{dx^2}$ at the point $(8, 80)$ is

- (A) 4 (B) 24 (C) 32 (D) 96 (E) 192

GO ON TO THE NEXT PAGE.



SCRATCH WORK

$$y' + xy = x$$

$$y(0) = -1$$

44. If y is a real-valued function defined on the real line and satisfying the initial value problem above, then $\lim_{x \rightarrow -\infty} y(x) =$

(A) 0 (B) 1 (C) -1 (D) ∞ (E) $-\infty$

45. How many positive numbers x satisfy the equation $\cos(97x) = x$?

(A) 1 (B) 15 (C) 31 (D) 49 (E) 96

46. A ladder 9 meters in length is leaning against a vertical wall on level ground. As the bottom end of the ladder is moved away from the wall at a constant rate of 2 meters per second, the top end slides downward along the wall. How fast, in meters per second, will the top end of the ladder be sliding downward at the moment the top end is 3 meters above the ground?

(A) $12\sqrt{2}$ (B) $6\sqrt{2}$ (C) $4\sqrt{2}$ (D) $\frac{1}{2\sqrt{2}}$ (E) $\frac{2}{3}$

GO ON TO THE NEXT PAGE.



SCRATCH WORK

47. The function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as follows.

$$f(x) = \begin{cases} 3x^2 & \text{if } x \in \mathbb{Q} \\ -5x^2 & \text{if } x \notin \mathbb{Q} \end{cases}$$

Which of the following is true?

- (A) f is discontinuous at all $x \in \mathbb{R}$.
 - (B) f is continuous only at $x = 0$ and differentiable only at $x = 0$.
 - (C) f is continuous only at $x = 0$ and nondifferentiable at all $x \in \mathbb{R}$.
 - (D) f is continuous at all $x \in \mathbb{Q}$ and nondifferentiable at all $x \in \mathbb{R}$.
 - (E) f is continuous at all $x \notin \mathbb{Q}$ and nondifferentiable at all $x \in \mathbb{R}$.
-

48. Let g be the function defined by $g(x, y, z) = 3x^2y + z$ for all real x , y , and z . Which of the following is the best approximation of the directional derivative of g at the point $(0, 0, \pi)$ in the direction of the vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$? (Note: \mathbf{i} , \mathbf{j} , and \mathbf{k} are the standard basis vectors in \mathbb{R}^3 .)

- (A) 0.2 (B) 0.8 (C) 1.4 (D) 2.0 (E) 2.6
-

49. What is the largest order of an element in the group of permutations of 5 objects?

- (A) 5 (B) 6 (C) 12 (D) 15 (E) 120
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

50. Let R be a ring and let U and V be (two-sided) ideals of R . Which of the following must also be ideals of R ?

I. $U + V = \{u + v : u \in U \text{ and } v \in V\}$

II. $U \cdot V = \{uv : u \in U \text{ and } v \in V\}$

III. $U \cap V$

- (A) II only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

51. Which of the following is an orthonormal basis for the column space of the real matrix $\begin{pmatrix} 1 & -1 & 2 & -3 \\ -1 & 1 & -3 & 2 \\ 2 & -2 & 5 & -5 \end{pmatrix}$?

(A) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

(B) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

(C) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{pmatrix} \right\}$

(D) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} \right\}$

(E) $\left\{ \begin{pmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \right\}$

GO ON TO THE NEXT PAGE.



SCRATCH WORK

52. A university's mathematics department has 10 professors and will offer 20 different courses next semester. Each professor will be assigned to teach exactly 2 of the courses, and each course will have exactly one professor assigned to teach it. If any professor can be assigned to teach any course, how many different complete assignments of the 10 professors to the 20 courses are possible?

(A) $\frac{20!}{2^{10}}$ (B) $\frac{10!}{2^9}$ (C) $10^{20} - 2^{10}$ (D) $10^{20} - 100$ (E) $\frac{20!10!}{2^{10}}$

53. Let f and g be functions of a real variable such that $g(x) = \int_0^x f(y)(y-x) dy$ for all x . If g is three times continuously differentiable, what is the greatest integer n for which f must be n times continuously differentiable?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

54. If a real number x is chosen at random in the interval $[0, 3]$ and a real number y is chosen at random in the interval $[0, 4]$, what is the probability that $x < y$?

(A) $\frac{1}{2}$ (B) $\frac{7}{12}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

GO ON TO THE NEXT PAGE.



SCRATCH WORK

55. If a and b are positive numbers, what is the value of $\int_0^{\infty} \frac{e^{ax} - e^{bx}}{(1 + e^{ax})(1 + e^{bx})} dx$?
- (A) 0 (B) 1 (C) $a - b$ (D) $(a - b)\log 2$ (E) $\frac{a - b}{ab}\log 2$
-

56. Which of the following statements are true?

- I. There exists a constant C such that $\log x \leq C\sqrt{x}$ for all $x \geq 1$.
- II. There exists a constant C such that $\sum_{k=1}^n k^2 \leq Cn^2$ for all integers $n \geq 1$.
- III. There exists a constant C such that $|\sin x - x| \leq C|x^3|$ for all real x .
- (A) None (B) I only (C) III only (D) I and III only (E) I, II, and III
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

57. For each positive integer n , let x_n be a real number in the open interval $\left(0, \frac{1}{n}\right)$. Which of the following statements must be true?
- I. $\lim_{n \rightarrow \infty} x_n = 0$
 - II. If f is a continuous real-valued function defined on $(0, 1)$, then $\{f(x_n)\}_{n=1}^{\infty}$ is a Cauchy sequence.
 - III. If g is a uniformly continuous real-valued function defined on $(0, 1)$, then $\lim_{n \rightarrow \infty} g(x_n)$ exists.
- (A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III
-

58. A circular helix in xyz -space has the following parametric equations, where $\theta \in \mathbb{R}$.

$$x(\theta) = 5 \cos \theta$$

$$y(\theta) = 5 \sin \theta$$

$$z(\theta) = \theta$$

Let $L(\theta)$ be the arc length of the helix from the point $P(\theta) = (x(\theta), y(\theta), z(\theta))$ to the point $(5, 0, 0)$, and let $D(\theta)$ be the distance between $P(\theta)$ and the origin. If $L(\theta_0) = 26$, then $D(\theta_0) =$

- (A) 6 (B) $\sqrt{51}$ (C) $\sqrt{52}$ (D) $14\sqrt{3}$ (E) $15\sqrt{3}$
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

59. Let A be a real 3×3 matrix. Which of the following conditions does NOT imply that A is invertible?

- (A) $-A$ is invertible.
- (B) There exists a positive integer k such that $\det(A^k) \neq 0$.
- (C) There exists a positive integer k such that $(I - A)^k = 0$, where I is the 3×3 identity matrix.
- (D) The set of all vectors of the form $A\mathbf{v}$, where $\mathbf{v} \in \mathbb{R}^3$, is \mathbb{R}^3 .
- (E) There exist 3 linearly independent vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^3$ such that $A\mathbf{v}_i \neq \mathbf{0}$ for each i .

60. A real-valued function f defined on \mathbb{R} has the following property.

For every positive number ϵ , there exists a positive number δ such that

$$|f(x) - f(1)| \geq \epsilon \text{ whenever } |x - 1| \geq \delta.$$

This property is equivalent to which of the following statements about f ?

- (A) f is continuous at $x = 1$.
- (B) f is discontinuous at $x = 1$.
- (C) f is unbounded.
- (D) $\lim_{|x| \rightarrow \infty} |f(x)| = \infty$
- (E) $\int_0^\infty |f(x)| dx = \infty$

GO ON TO THE NEXT PAGE.



SCRATCH WORK

61. A tank initially contains a salt solution of 3 grams of salt dissolved in 100 liters of water. A salt solution containing 0.02 grams of salt per liter of water is sprayed into the tank at a rate of 4 liters per minute. The sprayed solution is continually mixed with the salt solution in the tank, and the mixture flows out of the tank at a rate of 4 liters per minute. If the mixing is instantaneous, how many grams of salt are in the tank after 100 minutes have elapsed?

- (A) 2 (B) $2 - e^{-2}$ (C) $2 + e^{-2}$ (D) $2 - e^{-4}$ (E) $2 + e^{-4}$
-

62. Let S be the subset of \mathbb{R}^2 consisting of all points (x, y) in the unit square $[0, 1] \times [0, 1]$ for which x or y , or both, are irrational. With respect to the standard topology on \mathbb{R}^2 , S is

- (A) closed
(B) open
(C) connected
(D) totally disconnected
(E) compact
-

GO ON TO THE NEXT PAGE.



SCRATCH WORK

63. For any nonempty sets A and B of real numbers, let $A \cdot B$ be the set defined by

$$A \cdot B = \{xy : x \in A \text{ and } y \in B\}.$$

If A and B are nonempty bounded sets of real numbers and if $\sup(A) > \sup(B)$, then $\sup(A \cdot B) =$

- (A) $\sup(A) \sup(B)$
- (B) $\sup(A) \inf(B)$
- (C) $\max\{\sup(A) \sup(B), \inf(A) \inf(B)\}$
- (D) $\max\{\sup(A) \sup(B), \sup(A) \inf(B)\}$
- (E) $\max\{\sup(A) \sup(B), \inf(A) \sup(B), \inf(A) \inf(B)\}$

64. What is the value of the flux of the vector field \mathbf{F} , defined on \mathbb{R}^3 by $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, through the surface $z = \sqrt{1 - x^2 - y^2}$ oriented with upward-pointing normal vector field? (Note: \mathbf{i} , \mathbf{j} , and \mathbf{k} are the standard basis vectors in \mathbb{R}^3 .)

- (A) 0
- (B) $\frac{2\pi}{3}$
- (C) π
- (D) $\frac{4\pi}{3}$
- (E) 2π

GO ON TO THE NEXT PAGE.



SCRATCH WORK

65. Let g be a differentiable function of two real variables, and let f be the function of a complex variable z defined by

$$f(z) = e^x \sin y + ig(x, y),$$

where x and y are the real and imaginary parts of z , respectively. If f is an analytic function on the complex plane, then $g(3, 2) - g(1, 2) =$

- (A) e^2
(B) $e^2(\sin 3 - \sin 1)$
(C) $e^2(\cos 3 - \cos 1)$
(D) $e - e^3 \sin 2$
(E) $(e - e^3)\cos 2$
-

66. Let \mathbb{Z}_{17} be the ring of integers modulo 17, and let \mathbb{Z}_{17}^\times be the group of units of \mathbb{Z}_{17} under multiplication.

Which of the following are generators of \mathbb{Z}_{17}^\times ?

- I. 5
II. 8
III. 16
(A) None (B) I only (C) II only (D) III only (E) I, II, and III
-

S T O P

If you finish before time is called, you may check your work on this test.



SCRATCH WORK

NOTE: To ensure prompt processing of test results, it is important that you fill in the blanks exactly as directed.

SUBJECT TEST

- A. Print and sign your full name in this box:

| | | | |
|--------|--------|---------|----------|
| PRINT: | (LAST) | (FIRST) | (MIDDLE) |
| SIGN: | | | |

Copy this code in box 6 on your answer sheet. Then fill in the corresponding ovals exactly as shown.

| 6. TITLE CODE | | | | |
|---------------|---|---|---|---|
| 6 | 8 | 1 | 7 | 9 |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | ● | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 |
| ● | 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | ● | 7 |
| 8 | ● | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | ● |

Copy the Test Name and Form Code in box 7 on your answer sheet.

TEST NAME Mathematics

FORM CODE GR1268

GRADUATE RECORD EXAMINATIONS SUBJECT TEST

- B. The Subject Tests are intended to measure your achievement in a specialized field of study. Most of the questions are concerned with subject matter that is probably familiar to you, but some of the questions may refer to areas that you have not studied.

Your score will be determined by subtracting one-fourth the number of incorrect answers from the number of correct answers. Questions for which you mark no answer or more than one answer are not counted in scoring. If you have some knowledge of a question and are able to rule out one or more of the answer choices as incorrect, your chances of selecting the correct answer are improved, and answering such questions will likely improve your score. It is unlikely that pure guessing will raise your score; it may lower your score.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult for you. Go on to the other questions and come back to the difficult ones later if you can.

YOU MUST INDICATE ALL YOUR ANSWERS ON THE SEPARATE ANSWER SHEET. No credit will be given for anything written in this examination book, but you may write in the book as much as you wish to work out your answers. After you have decided on your response to a question, fill in the corresponding oval on the answer sheet. **BE SURE THAT EACH MARK IS DARK AND COMPLETELY FILLS THE OVAL.** Mark only one answer to each question. No credit will be given for multiple answers. Erase all stray marks. If you change an answer, be sure that all previous marks are erased completely. Incomplete erasures may be read as intended answers. Do not be concerned that the answer sheet provides spaces for more answers than there are questions in the test.

Example:

What city is the capital of France?

- (A) Rome
(B) Paris
(C) London
(D) Cairo
(E) Oslo

Sample Answer

(A) ● (C) (D) (E)

**CORRECT ANSWER
PROPERLY MARKED**

(A) ✗ (C) (D) (E)

(A) ✗ (C) (D) (E)

(A) ✗ (C) (D) (E)

(A) ✗ (C) (D) (E)

(A) ✗ (C) (D) (E)

IMPROPER MARKS

DO NOT OPEN YOUR TEST BOOK UNTIL YOU ARE TOLD TO DO SO.



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<https://ieltstep.com>

Worksheet for the GRE Mathematics Test, Form GR1268

Answer Key and Percentages* of Test Takers Answering Each Question Correctly

| QUESTION | | P+ | RESPONSE | |
|----------|--------|----|----------|---|
| Number | Answer | | C | I |
| 1 | E | 87 | | |
| 2 | C | 55 | | |
| 3 | D | 78 | | |
| 4 | A | 78 | | |
| 5 | E | 74 | | |
| 6 | C | 55 | | |
| 7 | C | 73 | | |
| 8 | B | 79 | | |
| 9 | A | 81 | | |
| 10 | A | 85 | | |
| 11 | B | 69 | | |
| 12 | B | 74 | | |
| 13 | D | 85 | | |
| 14 | B | 78 | | |
| 15 | A | 56 | | |
| 16 | B | 74 | | |
| 17 | B | 47 | | |
| 18 | A | 72 | | |
| 19 | E | 47 | | |
| 20 | E | 74 | | |
| 21 | B | 33 | | |
| 22 | C | 34 | | |
| 23 | D | 77 | | |
| 24 | E | 57 | | |
| 25 | A | 67 | | |
| 26 | D | 75 | | |
| 27 | D | 84 | | |
| 28 | D | 61 | | |
| 29 | C | 39 | | |
| 30 | A | 31 | | |
| 31 | C | 52 | | |
| 32 | E | 67 | | |
| 33 | C | 69 | | |
| 34 | B | 67 | | |
| 35 | B | 61 | | |

| QUESTION | | P+ | RESPONSE | |
|----------|--------|----|----------|---|
| Number | Answer | | C | I |
| 36 | C | 45 | | |
| 37 | C | 25 | | |
| 38 | C | 30 | | |
| 39 | D | 61 | | |
| 40 | C | 63 | | |
| 41 | A | 48 | | |
| 42 | E | 36 | | |
| 43 | A | 19 | | |
| 44 | B | 38 | | |
| 45 | C | 28 | | |
| 46 | C | 63 | | |
| 47 | B | 49 | | |
| 48 | B | 36 | | |
| 49 | B | 26 | | |
| 50 | D | 18 | | |
| 51 | E | 38 | | |
| 52 | A | 47 | | |
| 53 | A | 14 | | |
| 54 | C | 49 | | |
| 55 | E | 20 | | |
| 56 | D | 32 | | |
| 57 | C | 33 | | |
| 58 | B | 24 | | |
| 59 | E | 25 | | |
| 60 | D | 23 | | |
| 61 | E | 22 | | |
| 62 | C | 29 | | |
| 63 | E | 42 | | |
| 64 | E | 13 | | |
| 65 | E | 59 | | |
| 66 | B | 30 | | |

Total Correct (C): _____

Total Incorrect (I): _____

Raw Score: (C – I/4): _____

Scaled Score: _____

* The P+ column indicates the percent of GRE Mathematics Test examinees who answered each question correctly. It is based on a sample of October 2012 examinees selected to represent all GRE Mathematics Test examinees tested between July 1, 2011, and June 30, 2014.

Score Conversions for the GRE Mathematics Test, Form GR1268

| Raw Score | Scaled Score | Raw Score | Scaled Score |
|-----------|--------------|-----------|--------------|
| 63-66 | 910 | 30 | 640 |
| 60-62 | 900 | 29 | 630 |
| 58-59 | 890 | 28 | 620 |
| 56-57 | 880 | 27 | 610 |
| 54-55 | 870 | 26 | 600 |
| 53 | 860 | | |
| 52 | 850 | 25 | 590 |
| | | 24 | 580 |
| 51 | 840 | 23 | 570 |
| 50 | 830 | 21-22 | 560 |
| 49 | 820 | 20 | 550 |
| 48 | 810 | | |
| 47 | 800 | 19 | 540 |
| | | 18 | 530 |
| 46 | 790 | 17 | 520 |
| 45 | 780 | 16 | 510 |
| 44 | 770 | 14-15 | 500 |
| 43 | 760 | | |
| 42 | 750 | 13 | 490 |
| | | 12 | 480 |
| 41 | 740 | 11 | 470 |
| 40 | 730 | 10 | 460 |
| 39 | 720 | 8-9 | 450 |
| 38 | 710 | | |
| 37 | 700 | 7 | 440 |
| | | 6 | 430 |
| 36 | 690 | 4-5 | 420 |
| 35 | 680 | 3 | 410 |
| 34 | 670 | 2 | 400 |
| 32-33 | 660 | | |
| 31 | 650 | 0-1 | 390 |

SIDE 2

SUBJECT TEST

COMPLETE THE
CERTIFICATION STATEMENT,
THEN TURN ANSWER SHEET
OVER TO SIDE 1.

CERTIFICATION STATEMENT

Please write the following statement below, DO NOT PRINT.

"I certify that I am the person whose name appears on this answer sheet. I also agree not to disclose the contents of the test I am taking today to anyone."

Sign and date where indicated.

SIGNATURE: _____ DATE: _____
Month Day Year

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●. YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

| | | | | | | | |
|-----|---------------------|-----|---------------------|-----|---------------------|-----|---------------------|
| 115 | (A) (B) (C) (D) (E) | 147 | (A) (B) (C) (D) (E) | 179 | (A) (B) (C) (D) (E) | 211 | (A) (B) (C) (D) (E) |
| 116 | (A) (B) (C) (D) (E) | 148 | (A) (B) (C) (D) (E) | 180 | (A) (B) (C) (D) (E) | 212 | (A) (B) (C) (D) (E) |
| 117 | (A) (B) (C) (D) (E) | 149 | (A) (B) (C) (D) (E) | 181 | (A) (B) (C) (D) (E) | 213 | (A) (B) (C) (D) (E) |
| 118 | (A) (B) (C) (D) (E) | 150 | (A) (B) (C) (D) (E) | 182 | (A) (B) (C) (D) (E) | 214 | (A) (B) (C) (D) (E) |
| 119 | (A) (B) (C) (D) (E) | 151 | (A) (B) (C) (D) (E) | 183 | (A) (B) (C) (D) (E) | 215 | (A) (B) (C) (D) (E) |
| 120 | (A) (B) (C) (D) (E) | 152 | (A) (B) (C) (D) (E) | 184 | (A) (B) (C) (D) (E) | 216 | (A) (B) (C) (D) (E) |
| 121 | (A) (B) (C) (D) (E) | 153 | (A) (B) (C) (D) (E) | 185 | (A) (B) (C) (D) (E) | 217 | (A) (B) (C) (D) (E) |
| 122 | (A) (B) (C) (D) (E) | 154 | (A) (B) (C) (D) (E) | 186 | (A) (B) (C) (D) (E) | 218 | (A) (B) (C) (D) (E) |
| 123 | (A) (B) (C) (D) (E) | 155 | (A) (B) (C) (D) (E) | 187 | (A) (B) (C) (D) (E) | 219 | (A) (B) (C) (D) (E) |
| 124 | (A) (B) (C) (D) (E) | 156 | (A) (B) (C) (D) (E) | 188 | (A) (B) (C) (D) (E) | 220 | (A) (B) (C) (D) (E) |
| 125 | (A) (B) (C) (D) (E) | 157 | (A) (B) (C) (D) (E) | 189 | (A) (B) (C) (D) (E) | 221 | (A) (B) (C) (D) (E) |
| 126 | (A) (B) (C) (D) (E) | 158 | (A) (B) (C) (D) (E) | 190 | (A) (B) (C) (D) (E) | 222 | (A) (B) (C) (D) (E) |
| 127 | (A) (B) (C) (D) (E) | 159 | (A) (B) (C) (D) (E) | 191 | (A) (B) (C) (D) (E) | 223 | (A) (B) (C) (D) (E) |
| 128 | (A) (B) (C) (D) (E) | 160 | (A) (B) (C) (D) (E) | 192 | (A) (B) (C) (D) (E) | 224 | (A) (B) (C) (D) (E) |
| 129 | (A) (B) (C) (D) (E) | 161 | (A) (B) (C) (D) (E) | 193 | (A) (B) (C) (D) (E) | 225 | (A) (B) (C) (D) (E) |
| 130 | (A) (B) (C) (D) (E) | 162 | (A) (B) (C) (D) (E) | 194 | (A) (B) (C) (D) (E) | 226 | (A) (B) (C) (D) (E) |
| 131 | (A) (B) (C) (D) (E) | 163 | (A) (B) (C) (D) (E) | 195 | (A) (B) (C) (D) (E) | 227 | (A) (B) (C) (D) (E) |
| 132 | (A) (B) (C) (D) (E) | 164 | (A) (B) (C) (D) (E) | 196 | (A) (B) (C) (D) (E) | 228 | (A) (B) (C) (D) (E) |
| 133 | (A) (B) (C) (D) (E) | 165 | (A) (B) (C) (D) (E) | 197 | (A) (B) (C) (D) (E) | 229 | (A) (B) (C) (D) (E) |
| 134 | (A) (B) (C) (D) (E) | 166 | (A) (B) (C) (D) (E) | 198 | (A) (B) (C) (D) (E) | 230 | (A) (B) (C) (D) (E) |
| 135 | (A) (B) (C) (D) (E) | 167 | (A) (B) (C) (D) (E) | 199 | (A) (B) (C) (D) (E) | 231 | (A) (B) (C) (D) (E) |
| 136 | (A) (B) (C) (D) (E) | 168 | (A) (B) (C) (D) (E) | 200 | (A) (B) (C) (D) (E) | 232 | (A) (B) (C) (D) (E) |
| 137 | (A) (B) (C) (D) (E) | 169 | (A) (B) (C) (D) (E) | 201 | (A) (B) (C) (D) (E) | 233 | (A) (B) (C) (D) (E) |
| 138 | (A) (B) (C) (D) (E) | 170 | (A) (B) (C) (D) (E) | 202 | (A) (B) (C) (D) (E) | 234 | (A) (B) (C) (D) (E) |
| 139 | (A) (B) (C) (D) (E) | 171 | (A) (B) (C) (D) (E) | 203 | (A) (B) (C) (D) (E) | 235 | (A) (B) (C) (D) (E) |
| 140 | (A) (B) (C) (D) (E) | 172 | (A) (B) (C) (D) (E) | 204 | (A) (B) (C) (D) (E) | 236 | (A) (B) (C) (D) (E) |
| 141 | (A) (B) (C) (D) (E) | 173 | (A) (B) (C) (D) (E) | 205 | (A) (B) (C) (D) (E) | 237 | (A) (B) (C) (D) (E) |
| 142 | (A) (B) (C) (D) (E) | 174 | (A) (B) (C) (D) (E) | 206 | (A) (B) (C) (D) (E) | 238 | (A) (B) (C) (D) (E) |
| 143 | (A) (B) (C) (D) (E) | 175 | (A) (B) (C) (D) (E) | 207 | (A) (B) (C) (D) (E) | 239 | (A) (B) (C) (D) (E) |
| 144 | (A) (B) (C) (D) (E) | 176 | (A) (B) (C) (D) (E) | 208 | (A) (B) (C) (D) (E) | 240 | (A) (B) (C) (D) (E) |
| 145 | (A) (B) (C) (D) (E) | 177 | (A) (B) (C) (D) (E) | 209 | (A) (B) (C) (D) (E) | 241 | (A) (B) (C) (D) (E) |
| 146 | (A) (B) (C) (D) (E) | 178 | (A) (B) (C) (D) (E) | 210 | (A) (B) (C) (D) (E) | 242 | (A) (B) (C) (D) (E) |

| TR | TW | TFS | TCS | 1R | 1W | 1FS | 1CS | 2R | 2W | 2FS | 2CS |
|------------------|----|-----|-----|----|----|-----|-----|----|----|-----|-----|
| FOR ETS USE ONLY | | | | 3R | 3W | 3FS | 3CS | 4R | 4W | 4FS | 4CS |
| | | | | 5R | 5W | 5FS | 5CS | 6R | 6W | 6FS | 6CS |

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